NAME AND SURNAME : ....

MATHEMATICS TEACHER : ....

Hudson Park High School

GRADE 12
MATHEMATICS
June Paper 1

Time	:	3 hour	Date	:	26 May 2014
Marks	:	150	Examiner	:	SLT
			Moderator(s)	:	SLK and CLM

INSTRUCTIONS

1. Illegible work, in the opinion of the marker, will earn zero marks.
2. Number your answers clearly and accurately, exactly as they appear on the question paper.
3. NB **Start each question at the top of a page.**
4. NB
   - **Staple your foolscap answers and answer sheets (in the correct order)**
   - **Fill in the details requested on the front of the question paper and hand your question paper in separately.**
5. Employ relevant formulae and show all working out. Answers alone may not be awarded full marks.
6. (Non programmable and non graphical) Calculators may be used, unless their usage is specifically prohibited.
7. Round off answers to 2 decimal places, where necessary, unless instructed otherwise.

QUESTION 1 [28 marks]

1.1. Solve for x :

1.1.1. $5x^2 - 4x - 11 = 0$ 3

1.1.2. $0 > -2(2x-5)(x+3)$ 3

1.1.3.. $-7x^{-\frac{3}{4}} + 6x^{-\frac{3}{2}} - 3 = 0$ 5 (11)

1.2. Solve for $\frac{x}{y}$: $3x^2 + 11xy - 4y^2 = 0$. (3)

1.3. Solve for x and y :

$$3y^2 - x^2 - 2xy = 3$$
$$4y - x = -6$$

(7)

1.4. Simplify fully, without the use of a calculator :

1.4.1. $\frac{9^{1006}}{3^{2015} + 3^{2013}}$ 3

1.4.2. $\left(\sqrt[3]{54} - \sqrt[3]{250} \right)^3$ 4 (7)

QUESTION 2 [11 marks]

2.1. Given : $x^2 + bx - 2 + k(x^2 + 3x + 2) = 0$

2.1.1. For the given equation, show that the discriminant will be

$$\Delta = k^2 + 6bk + b^2 + 8$$ 3

2.1.2. If $b = 0$, discuss the nature of the roots of the given equation. 3 (6)

2.2. For which value(s) of c will the following two graphs

$$f(x) = -2x^2 - x + 3$$

$$g(x) = 3x + c$$

be tangential ? (5)

QUESTION 3 [16 marks]

3.1. Given : $3 - \frac{3}{2} + \frac{3}{4} - \frac{3}{8} \cdots$

For this series, calculate :

3.1.1. T_{20} 3

3.1.2. S_{15} 2 (5)

3.2. Given : $-\frac{1}{2}\ ;\ 10\ ;\ \frac{1}{2}\ ;\ 10\ ;\ \frac{3}{2}\ ;\ 10\ ;\ \frac{5}{2}\ ;\ 10\ ;\ \cdots$

For this sequence, determine :

3.2.1. T_{100} 1

3.2.2. S_{99} 5 (6)

3.3. Pupils are investigating the sequence

$18\ ;20\ ;22\ ;24\ ;\ \cdots;\ 5000$

to determine how many terms (of the sequence) are divisible by 3.

Two pupils make suggestions :

Pupil 1 : Write down the first 3 terms in the given sequence that are divisible by 3

Pupil 2 : Find the last term in the given sequence that is divisible by 3.

Now, use the pupils' suggestions and determine how many terms in the given sequence will be divisible by 3. (5)

QUESTION 4 [9 marks]

4.1. By involving infinite geometric series, write $3,\dot{7}\dot{5}$ as an improper fraction. (4)

4.2. Given : $\sum_{k=3}^{\infty} (1-2x)^k$

4.2.1. Write down the first 3 terms of the given series, in terms of x.
You do NOT need to
- multiply out, or
- simplify

the terms. 1

4.2.2. Calculate the value(s) of x for which the given series will converge. 4 (5)

QUESTION 5 [4 marks]

5. Given : $-10\,;\,-6\,;4\,;20\,;\,\cdots$

Determine an expression for the general term, T_n, of the given sequence. (4)

QUESTION 6 [16 marks]

6.1. Sketch a rough graph of :

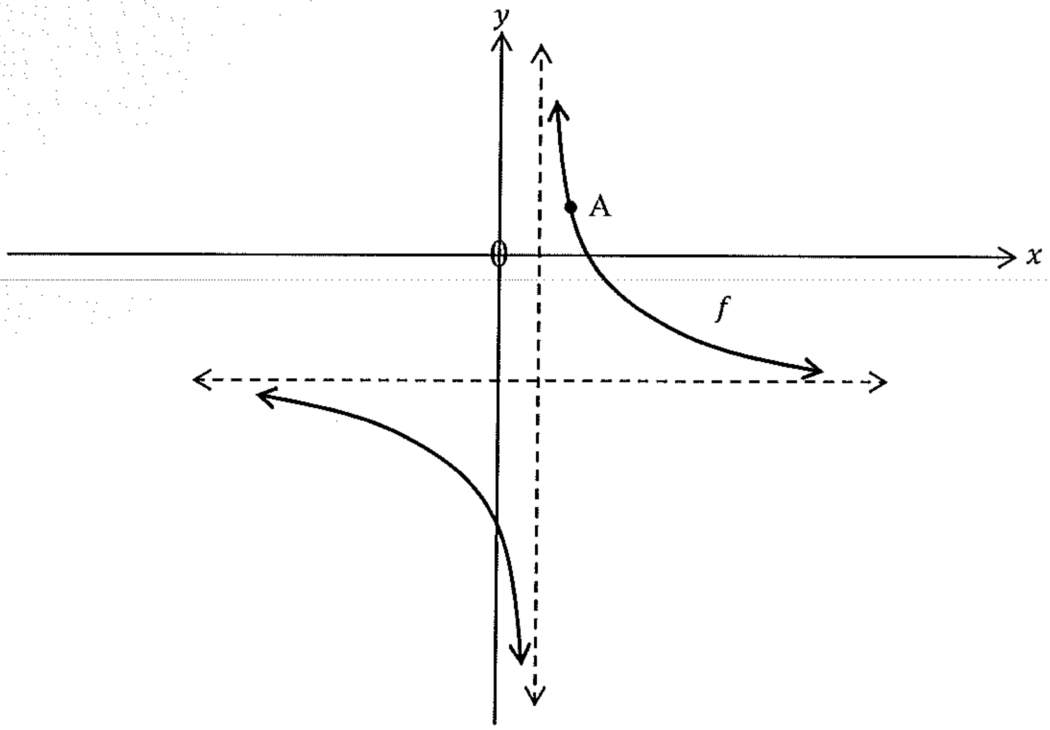
$$g(x) = -3^{x+1} + 9$$

Show all relevant details on your diagram. (5)

6.2. Sketched below is a rough graph of the hyperbola

$$f(x) = \frac{5+ax}{x+b}$$

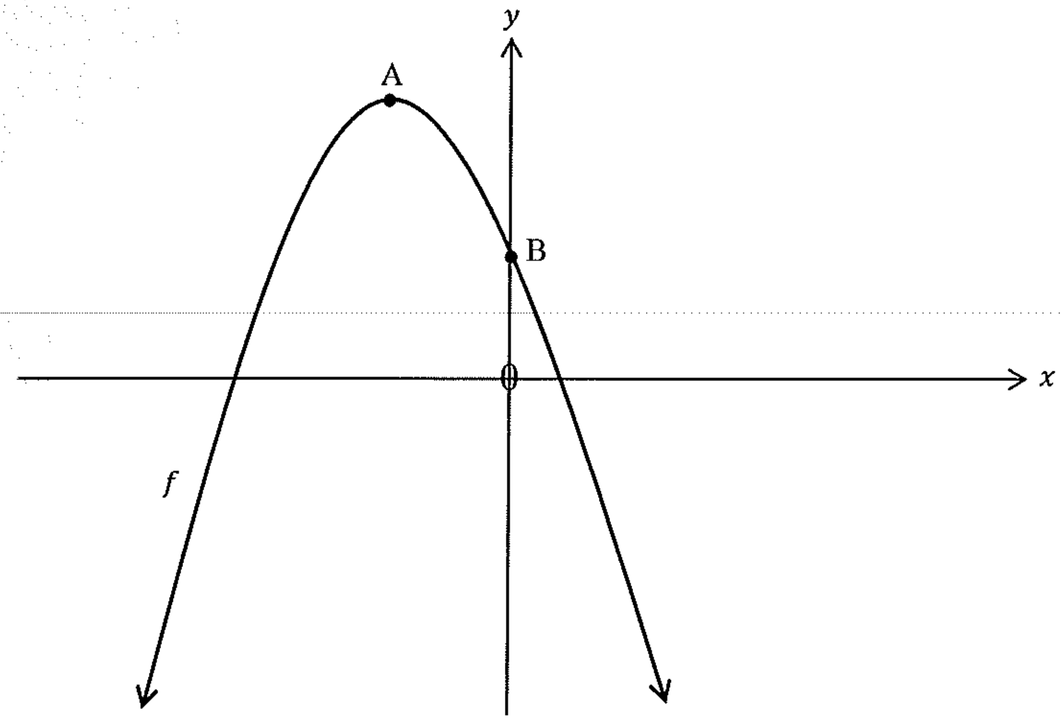
The equation of the vertical asymptote of f is $x = 1$ and point A(2 ; 1) lies on the hyperbola :



6.2.1.	Calculate the values of a and b.		3	
6.2.2.	Hence, show that the equation of f can be written as $f(x) = \frac{3}{x-1} - 2$		2	
6.2.3.	Write down the coordinates of B, if B is the reflection of A in the horizontal asymptote of f.		2	
6.2.4.1.	State the equation of the axis of symmetry of f that has a negative gradient.	2		
6.2.4.2.	Write down the coordinates of C, if C is the reflection of A in the line stated in (6.2.4.1.).	2	4	(11)

QUESTION 7 [11 marks]

7. A parabola, f, is shown below. $A\left(-\frac{5}{4}\,;\,\frac{49}{8}\right)$ is the turning point of the parabola and $B(0\,;3)$ is a point on the parabola :



7.1. Determine the equation of f, showing that it will be

$$f(x) = -2x^2 - 5x + 3$$ (4)

7.2. For which value(s) of k, will

$$2x^2 + 5x - 3 = 2k$$

have 2 distinct real roots, with both of those roots being negative ? (4)

7.3.1. Will f^{-1}, the inverse of f, be a function ? 1

7.3.2. Give a reason for your answer to (7.3.1.), with reference to :

7.3.2.1. f 1

7.3.2.2. f^{-1} 1 2 (3)

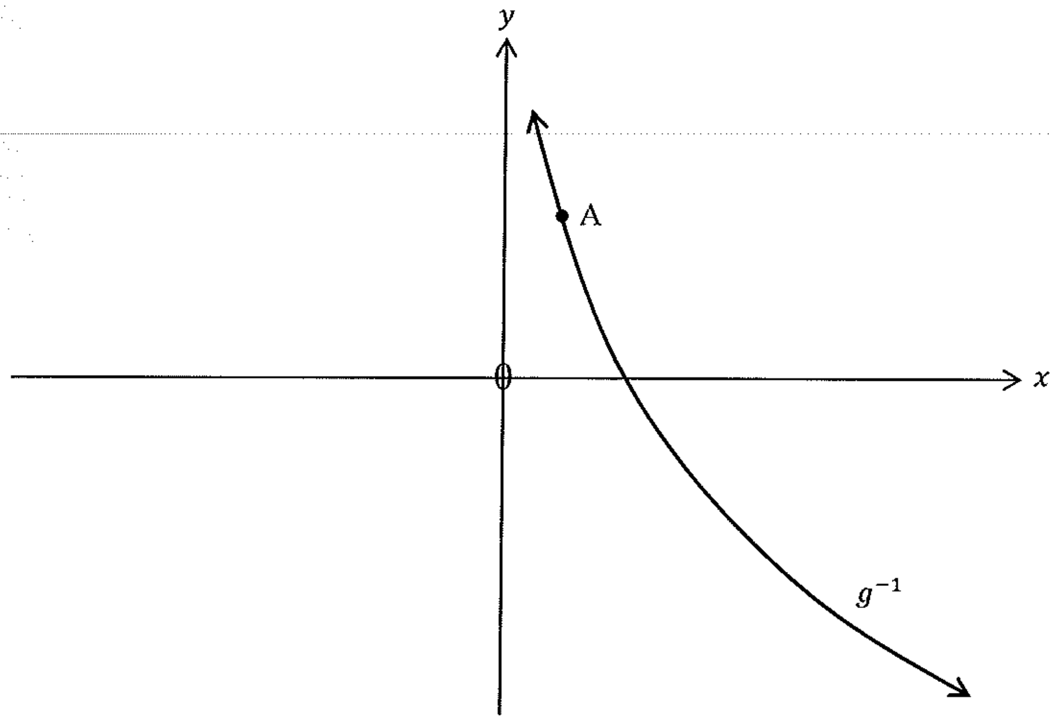
QUESTION 8 [10 marks]

USE THE ANSWER SHEET PROVIDED

8. The inverse of g

$$g^{-1}(x) = -\log_a x$$

where $a > 0$, and A(0,5 ; 0,63) are shown below :



8.1. Calculate the value of a. (4)

8.2. Solve for x: $-\log_a x \geq 0{,}63$ (2)

8.3. On the given set of axes, sketch the graph of g. (2)

8.4. State the domain of h, if

$$h(x) = g^{-1}(x-2)$$ (2)

QUESTION 9 [19 marks]

9.1. Convert a nominal interest rate of 9 % per annum compounded quarterly to an effective annual interest rate, as a percentage. (2)

9.2. A vehicle depreciates, according to the diminishing balance method, BY two thirds of its value, in 8 years. Calculate the vehicle's rate of depreciation, as a percentage. (4)

9.3. R 300 payments are made, at the end of each month, for exactly k years, into an account that earns 5 % interest per annum compounded monthly. If the amount saved is R 11 626,00 , calculate the value of k . (6)

9.4. A house is on the market for R 1 500 000,00. You put down a 20 % cash deposit and take out a 20 year loan for the balance. The banks charges interest at a rate of 8 % per annum compounded monthly. You make your first repayment one month after the loan is granted.

9.4.1. How much money did you loan from the bank ? 1

9.4.2. Calculate your monthly repayments. 3

9.4.3. After 15 years, how much will you still owe the bank ? 3 (7)

QUESTION 10 [15 marks]

10.1. Given : $f(x) = -x^2 + 3$

Determine $f'(x)$ from first principles. (5)

10.2. Calculate the average gradient of $i(x) = -\frac{3}{x} + 1$ between $x = -1$ and $x = 2$. (3)

10.3. When $g(x) = 3x^4 + ax - 5$ is divided by $x + 2$ the remainder is 3 . Calculate the value of a . (3)

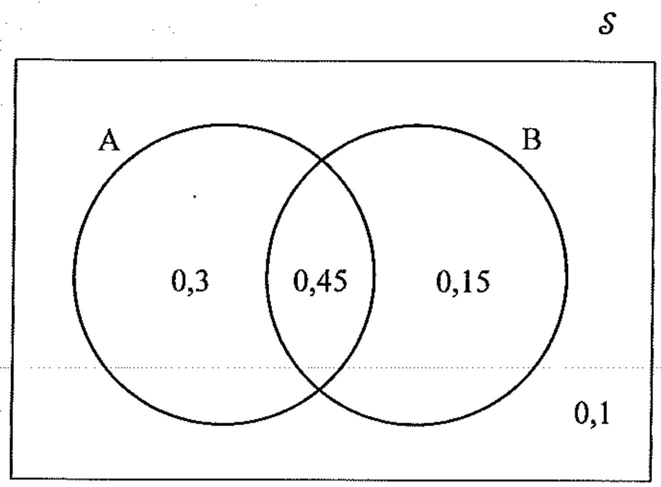
10.4. Given : $h(x) = 12x^3 - 17x^2 + 36x - 20$

10.4.1. Use the factor theorem to show that $3x - 2$ is a factor of h . 2

10.4.2. Now, determine the other factor. 2 (4)

QUESTION 11 [11 marks]

11.1. Events A and B are represented in the probability Venn Diagram below :

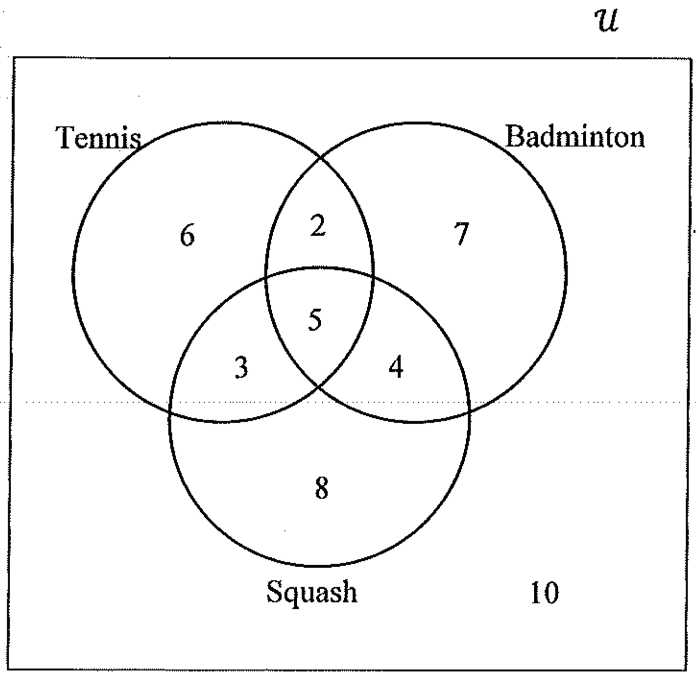


11.1. Are events A and B :

11.1.1 mutually exclusive ? Justify your answer. 2

11.1.2. independent ? Justify your answer. 2 (4)

11.2. The Venn Diagram below represents the racquet sports involvement of pupils at a school :



How many pupils play at least two sports ? (1)

11.3. Two identical bags, X and Y, have marbles in them.

Bag X contains 3 blue marbles and 5 red marbles.
Bag Y contains 4 blue marbles and 2 red marbles.

A bag is chosen and then a marble is chosen from that bag.

11.3.1. Represent the information as a fully labeled tree diagram. Probabilities and outcomes should be shown appropriately. 3

11.3.2. What is the probability that a blue marble will be chosen ? 3 (6)

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r-1}$; $\quad r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

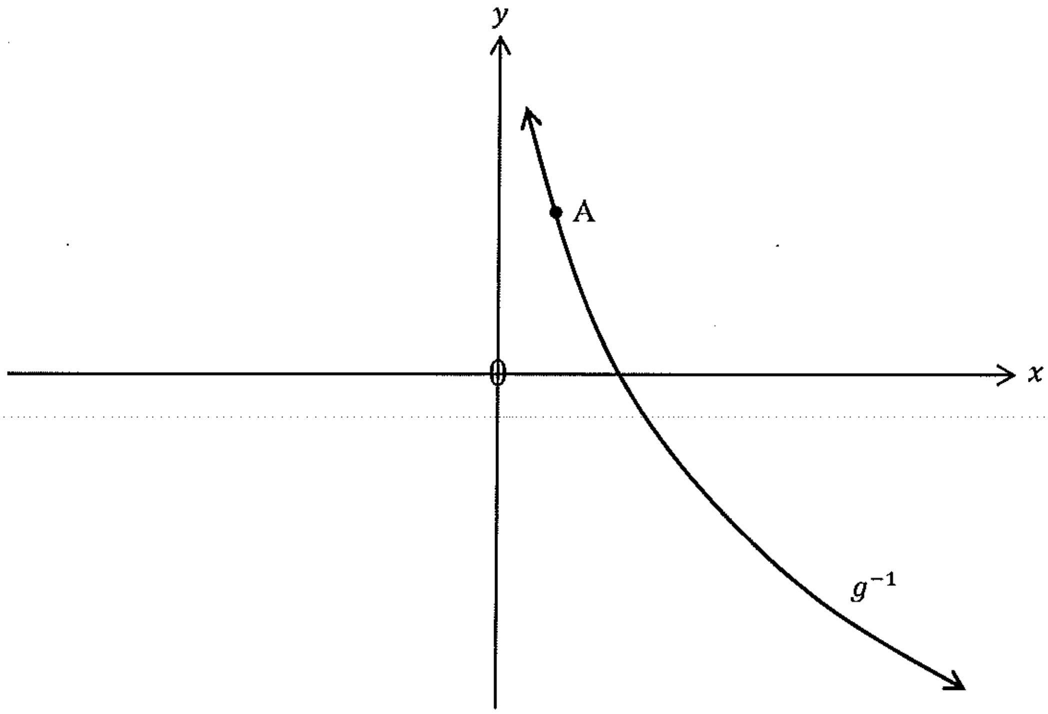
$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

ANSWER SHEET FOR QUESTION 8

8.



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